Distributed Control of Inverter-Based Power Grids

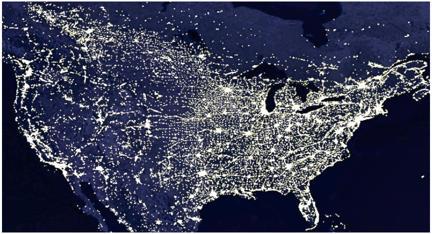
John W. Simpson-Porco

ESIF Workshop: Frontiers in Distributed Optimization & Control of Sustainable Power Systems

Co-Authors: Florian Dörfler (ETH) and Francesco Bullo (UCSB)

January 26th, 2016

Electricity & The Power Grid



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- **Hierarchical** grid: generate/transmit/consume
- **Challenges**: multi-scale, need reliability + performance

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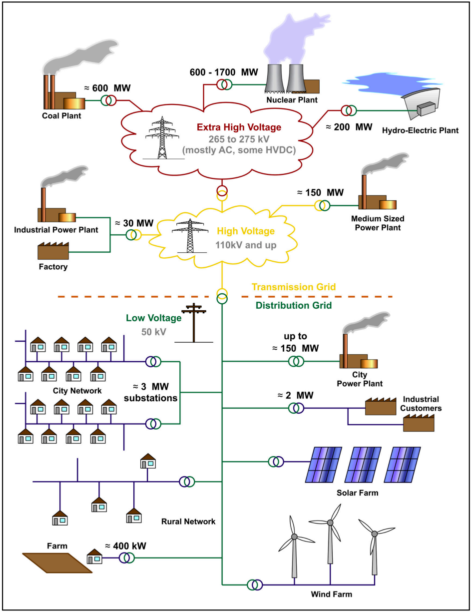

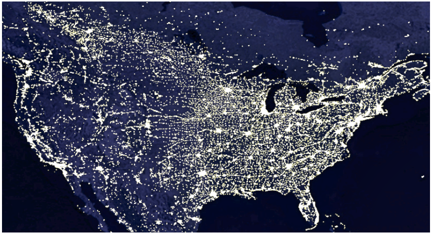

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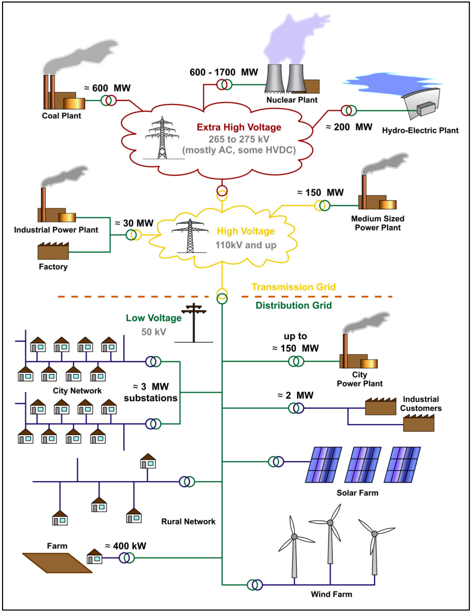

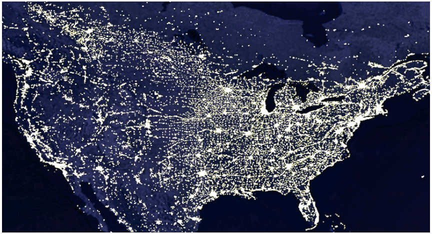

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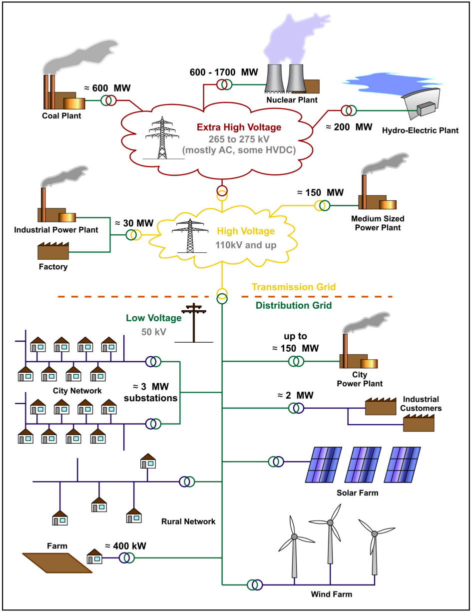

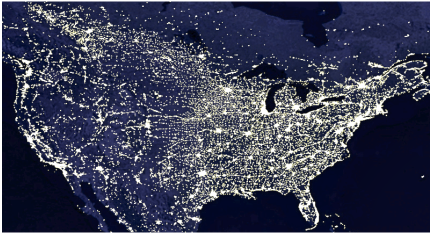

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What are the control strategies?

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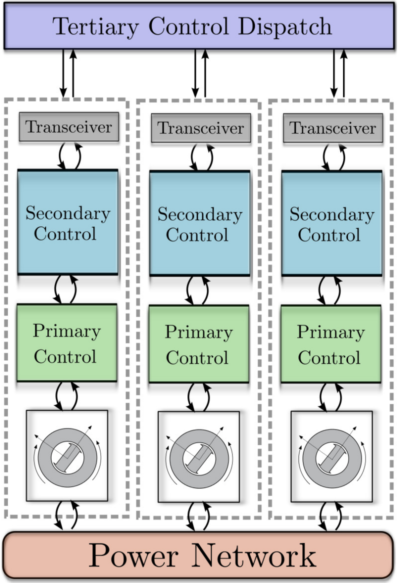
Bulk Power System Control Architecture & Objectives

Hierarchy by spatial/temporal scales and physics



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast

2. **Secondary control** (minutes)
   - Goal: restore frequency
   - Strategy: centralized

1. **Primary control** (real-time)
   - Goal: stabilize freq. and volt.
   - Strategy: decentralized

Q: Is this hierarchical architecture still appropriate for new applications?

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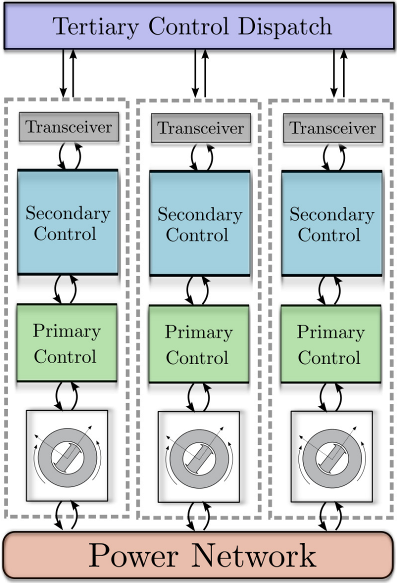

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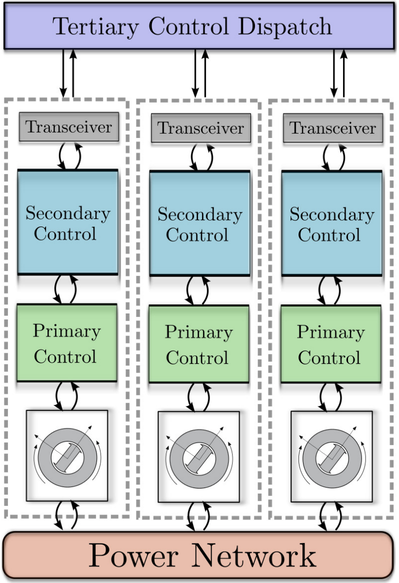

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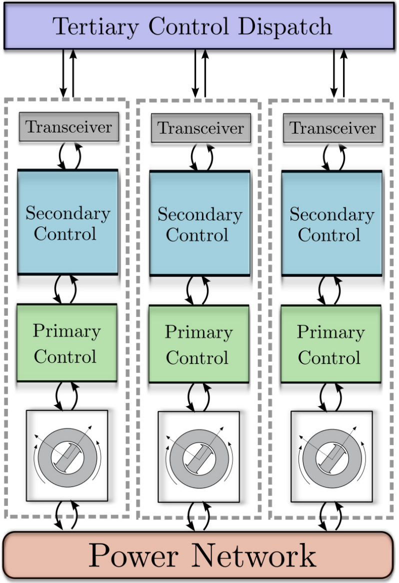

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Two Major Trends

(New York Magazine)

Trend 1: Physical Volatility

1. ~~bulk~~ distributed generation, regulation (33 by 2020 in CA, GEA in ON)
2. growing demand & old infrastructure

⇒ lowered inertia & robustness margins

Trend 2: Technological Advances

1. sensors, actuators & grid-edge resources (PMUs, FACTS, flexible loads)
2. control of cyber-physical systems

⇒ cyber-coordination layer for smart grid

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(Electronic Component News)

Outline

Relevant Publications

J. W. Simpson-Porco, F. Dörfler, and F. Bullo. Voltage stabilization in microgrids via quadratic droop control. *IEEE Transactions on Automatic Control*, May 2015. Note: Conditionally accepted.

J. W. Simpson-Porco, F. Dörfler, and F. Bullo. Voltage Collapse in Complex Power Grids. February 2015. Note: Accepted.

J. W. Simpson-Porco, Q. Shafiee, F. Dörfler, J. C. Vasquez, J. M. Guerrero, and F. Bullo. Secondary Frequency and Voltage Control in Islanded Microgrids via Distributed Averaging. *IEEE Transactions on Industrial Electronics*, 62(11):7025-7038, 2015.

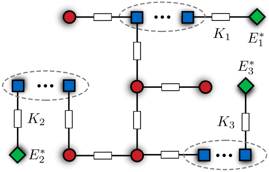
F. Dörfler, J. W. Simpson-Porco, and F. Bullo. Breaking the Hierarchy: Distributed Control & Economic Optimality in Microgrids. *IEEE Transactions on Control of Network Systems*. Note: To Appear.

J. W. Simpson-Porco, F. Dörfler, and F. Bullo. Synchronization and Power-Sharing for Droop-Controlled Inverters in Islanded Microgrids. *Automatica*, 49(9):2603-2611, 2013.

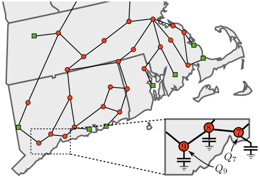
Research supported by NSF NSERC CRSNG

Project Samples: Voltage Control/Collapse

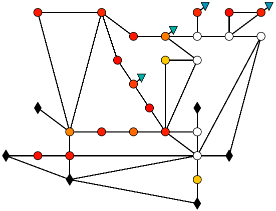
Quadratic Droop Control (TAC)



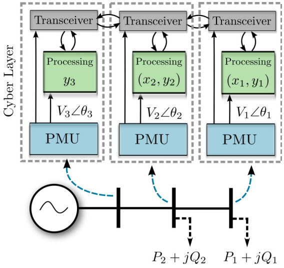
Voltage Collapse (Nat. Comms.)



Optimal Distrib. Volt/Var (CDC)



Collapse W.A.M. (TSG)



Outline

Microgrids

Structure

- low-voltage, small footprint
- grid-connected or islanded
- autonomously managed

Applications

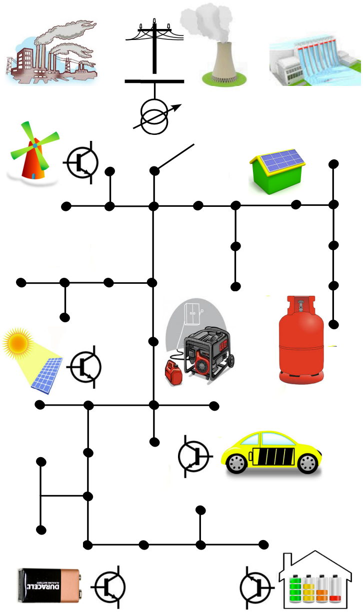
- hospitals, military, campuses, large vehicles, & isolated communities

Benefits

- naturally distributed for renewables
- scalable, efficient & redundant

Operational challenges

- low inertia & uncertainty
- plug'n'play & no central authority



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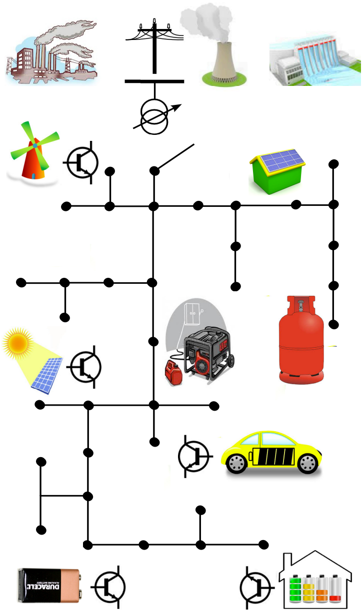
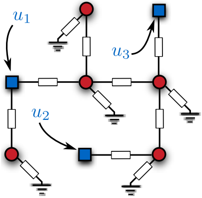


Modeling I: AC circuits

1. **Loads (●) and Inverters (■)**
2. **Quasi-Synchronous:** $\omega \simeq \omega^* \Rightarrow V_i = E_i e^{j\theta_i}$
3. **Load Model:** Constant powers P_i^*, Q_i^*
4. **Coupling Laws:** Kirchoff and Ohm: $Y_{ij} = G_{ij} + jB_{ij}$
5. **Line Characteristics:** $G_{ij}/B_{ij} =$ const. (today, lossless $G_{ij} = 0$)
6. **Decoupling:** $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ (normal operating conditions)



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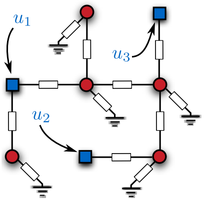

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- active power: $P_i \;=\; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

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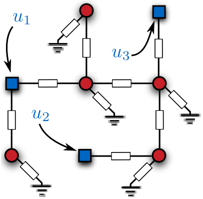

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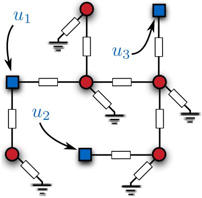

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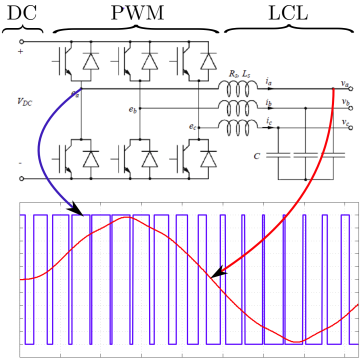
- trigonometric active power flow: $P_i(\theta) \;=\; \sum_j B_{ij} \sin(\theta_i - \theta_j)$
- quadratic reactive power flow: $Q_i(E) \;=\; -\sum_j B_{ij} E_i E_j$

Modeling II: Inverter-interfaced sources

also applies to frequency-responsive loads

Power **inverters** are ...

- **interface** between AC grid and DC or variable AC sources
- operated as **controllable** ideal voltage sources



Assumptions:

- Fast, stable inner/outer loops (voltage/current/impedance)
- Good harmonic filtering
- Balanced 3-phase operation

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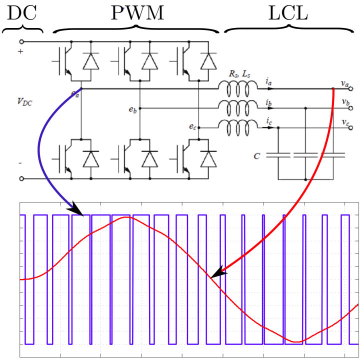
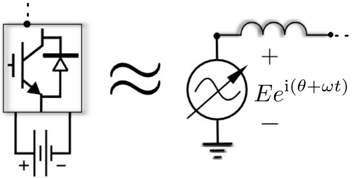
$$\omega_i = u_i^{\text{freq}}\,, \qquad \tau_i \dot{E}_i = u_i^{\text{volt}}$$





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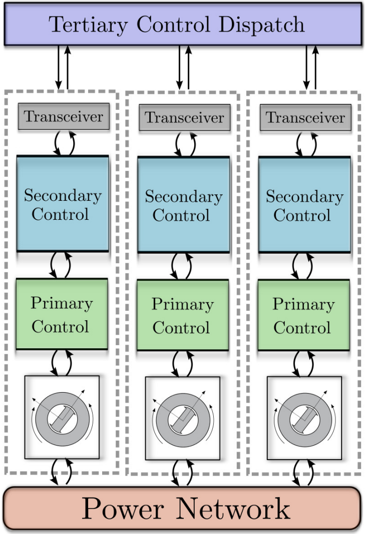
Tertiary Control Dispatch
Transceiver
Transceiver
Transceiver
Secondary Control
Secondary Control
Secondary Control
Primary Control
Primary Control
Primary Control
Power Network

Open-Loop System & Control Objectives

Frequency Open-Loop	**Voltage Open-Loop**
Inverter Dynamics ($i \in \mathcal{I}$):	Inverter Dynamics ($i \in \mathcal{I}$):
$\omega_i = \dot{\theta}_i = u_i^{\text{freq}}$ $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$	$\tau_i \dot{E}_i = u_i^{\text{volt}}$ $Q_i(E) = -\sum_j B_{ij} E_i E_j$
Power Balance ($i \in \mathcal{L}$):	Power Balance: ($i \in \mathcal{L}$)
$0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$	$0 = Q_i^* + \sum_j B_{ij} E_i E_j$

Primary Control Objectives:

1. **Stabilization:** Ensure stable frequency/voltage dynamics
2. **Balance:** Balance supply/demand for variable loads
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Primary Droop Control

“Grid-forming” decentralized control

Key Idea: emulate generator speed & AVR control

Primary Droop Control

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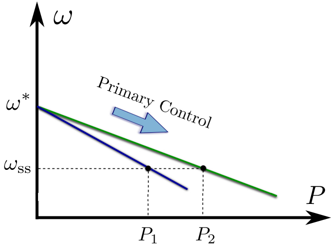
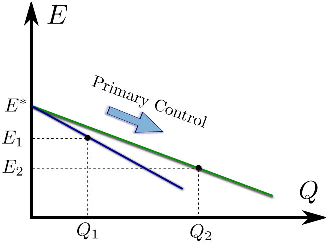
Key Idea: emulate generator speed & AVR control

Frequency Droop Control	**Voltage Droop Control**
$\omega_i = \omega^* - m_i P_i(\theta)$	$\tau_i \dot{E}_i = -(E_i - E^*) - n_i Q_i(E)$





Primary Droop Control

"Grid-forming" decentralized control

Key Idea: emulate generator speed & AVR control

Frequency Droop Control	**Quad. Voltage Droop Control**
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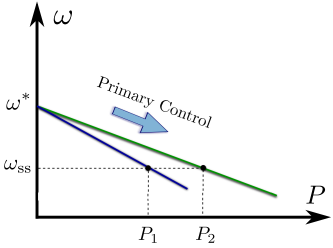

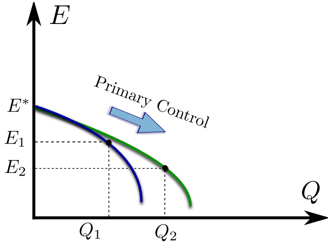

Spring Network Interpretations of Equilibria

Frequency Droop Control	**Voltage Droop Control**
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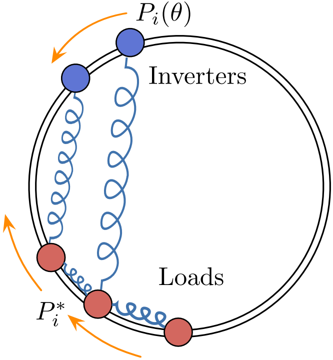

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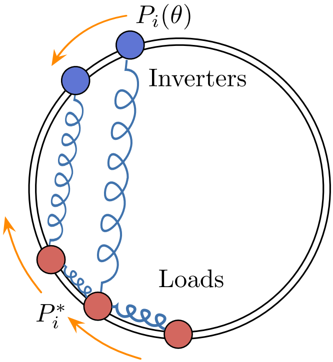

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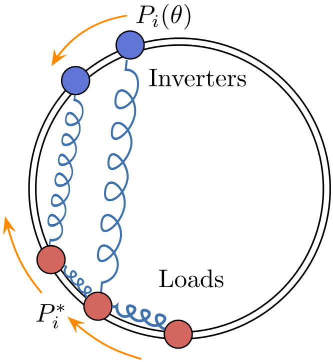

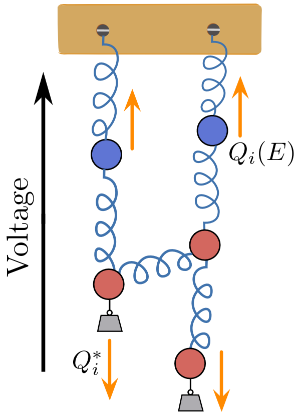

Droop Control Stability Conditions

Frequency Droop Control	**Voltage Droop Control**
$0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$ $\dot{\theta}_i = -m_i \sum_j B_{ij} \sin(\theta_i - \theta_j)$	$0 = Q_i^* + \sum_j B_{ij} E_i E_j$ $\tau_i \dot{E}_i = -E_i(E_i - E^*) + n_i \sum_j B_{ij} E_i E_j$
Theorem: Frequency Stability (JWSP, FD, & FB '12) ∃! loc. exp. stable angle equilibrium θ_{eq} iff $\boxed{\frac{(A^\dagger P)_{ij}}{B_{ij}} < 1}$ for all edges (i, j) of microgrid.	**Theorem: Voltage Stability** (JWSP, FD, & FB '15) ∃! loc. exp. stable voltage equilibrium point E_{eq} if $\boxed{\frac{4}{(E^*)^2}(B_{LL}^{-1} Q_L)_i < 1}$ for all load nodes i of microgrid.

Necessary and Sufficient

Tight and Sufficient

Droop Control Stability Conditions

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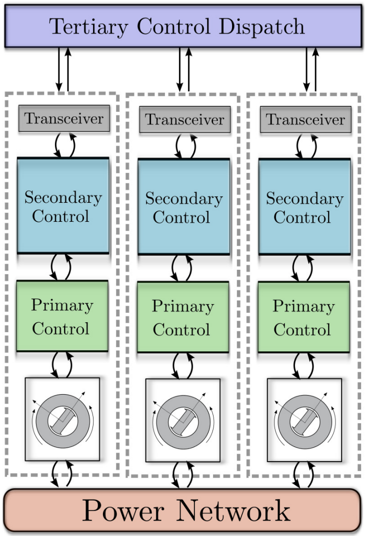
Tight and Sufficient

Open Primary Control Problems

1. Coupled equilibrium and stability analysis
2. New controllers for $G_{ij}/B_{ij} \neq$ constant
3. Basins of attraction
4. Limits of decentralized control



Tertiary Control Dispatch
Transceiver
Secondary Control
Primary Control
Transceiver
Secondary Control
Primary Control
Transceiver
Secondary Control
Primary Control
Power Network

Economic dispatch

minimize the total cost of generation

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minimize the total cost of generation

$$\text{minimize}_{\theta \in \mathbb{T}^n} \quad f(\theta) = \frac{1}{2} \sum_{\text{inverters}} \alpha_i [P_i(\theta)]^2$$

subject to

load power balance: $0 = P_i^* - P_i(\theta)$

branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

inverter injection constraints: $P_i(\theta) \in \left[0, \overline{P}_i\right]$

Variations: general strictly convex & differentiable cost.

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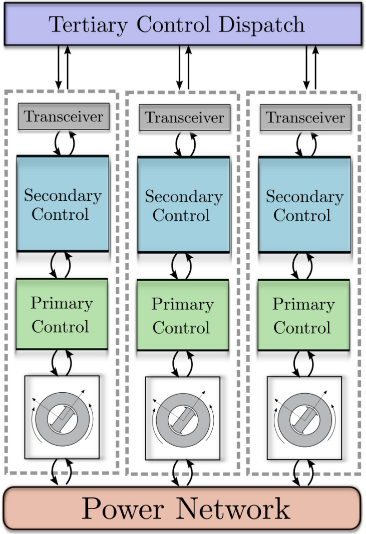
Result: Droop = **decentralized primal algorithm** for this problem.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Secondary frequency control in power networks

Problem: steady-state frequency deviation ($\omega_{\mathrm{ss}} = \omega^*$)

Solution: integral control on frequency error

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Interconnected Systems	Isolated Systems
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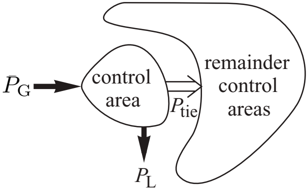

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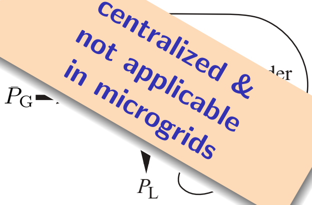

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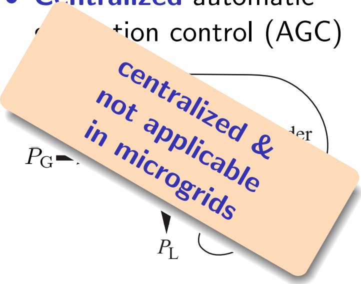

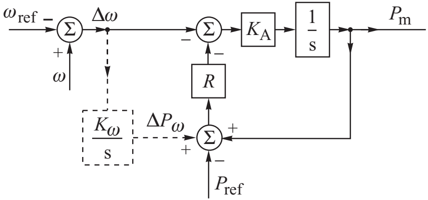

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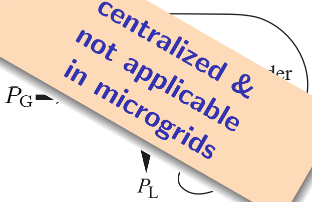

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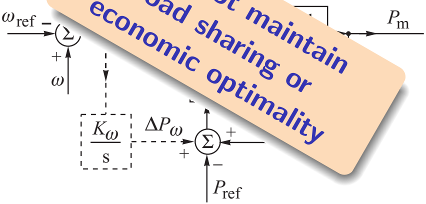

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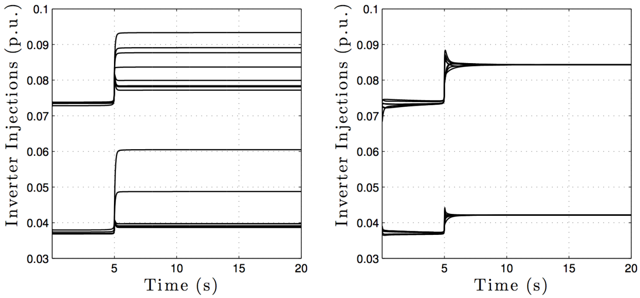
Interconnected Systems | Isolated Systems



(a) Decentralized control   (b) Centralized control

P_m

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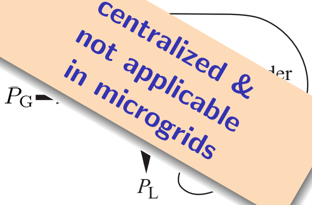

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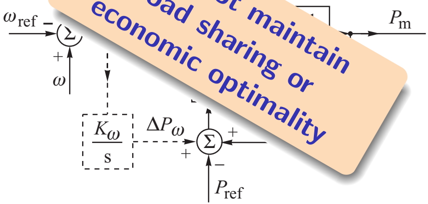

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What about **distributed** secondary control strategies?

Distributed Averaging PI (DAPI) Frequency Control

$$\omega_i = \omega^* - m_i P_i(\theta) - \Omega_i$$

$$k_i \dot{\Omega}_i = (\omega_i - \omega^*) - \sum_{j \subseteq \text{inverters}} a_{ij} \cdot (\Omega_i - \Omega_j)$$

1. no tuning, no model dependence
2. weak comm. requirements
3. maintains load sharing
   (share burden of sec. control)

Simple & Intuitive

Theorem: Stability of DAPI
[JWSP, FD, & FB, '13]

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$\Updownarrow$
Droop-Controlled System Stable

(grid-conscious sec. control)

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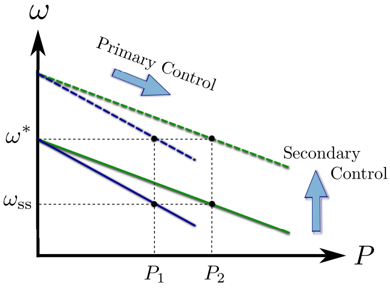

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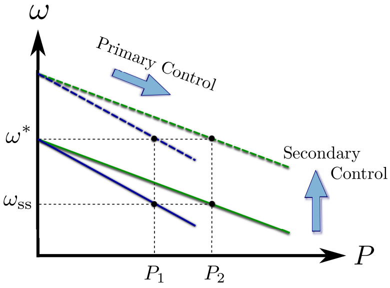

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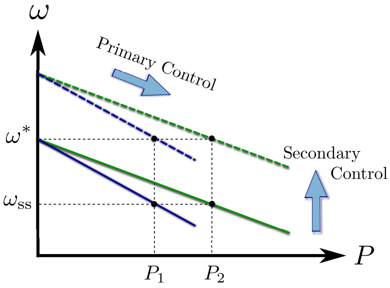

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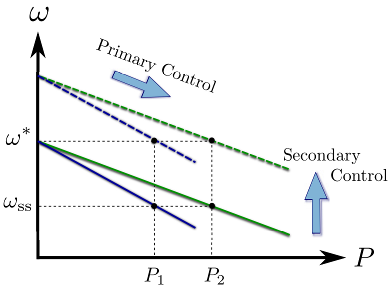

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Problem: steady-state voltage deviations ($E_i = E_i^*$)

Goals: Voltage regulation $E_i \to E_i^*$, "load" sharing $Q_i/Q_i^* = Q_j/Q_j^*$

Bad News: These goals are *fundamentally* conflicting.

We propose a **heuristic compromise**.

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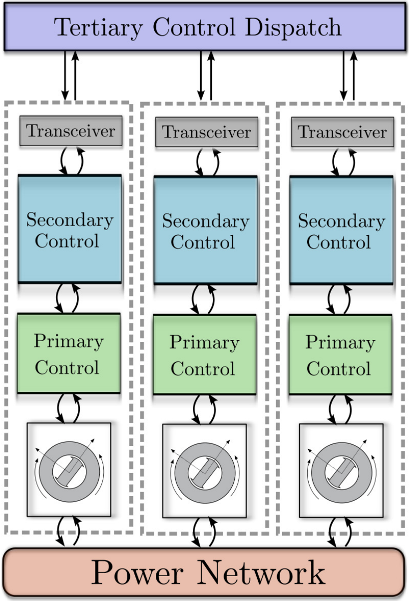
From Hierarchical Control to DAPI Control

flat hierarchy, distributed, no time-scale separations, & model-free



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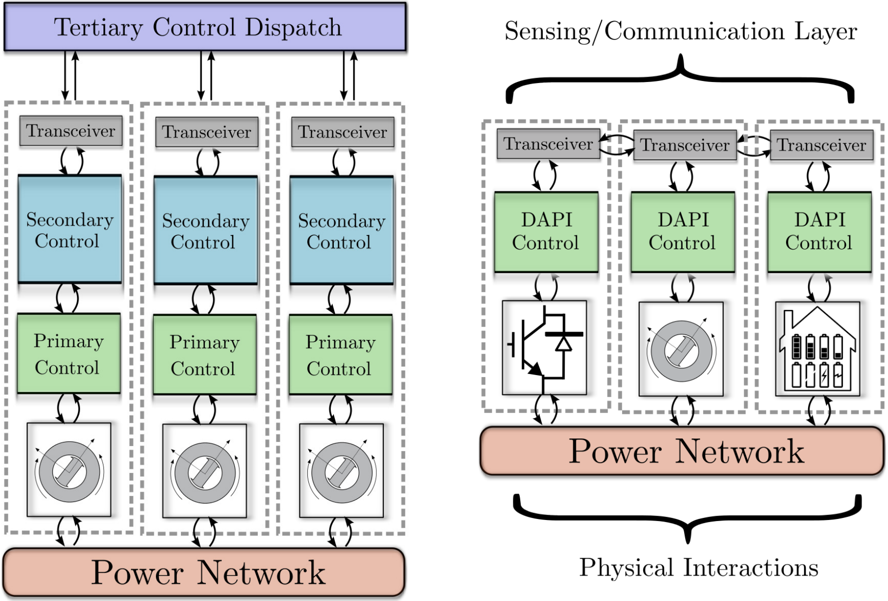
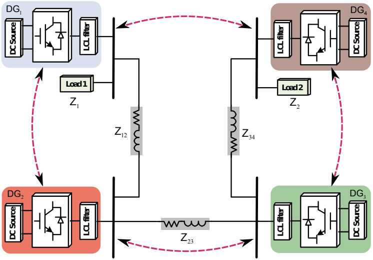


Experimental Validation of DAPI Control

Experiments @ Aalborg University Intelligent Microgrid Laboratory



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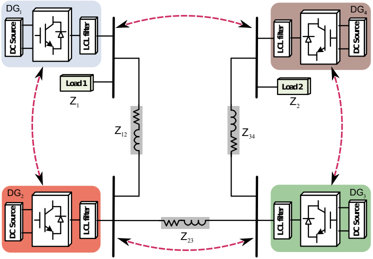

1. $t < 7$: Droop Control
2. $t = 7$: DAPI Control
3. $t = 22$: Remove Load 2
4. $t = 36$: Attach Load 2

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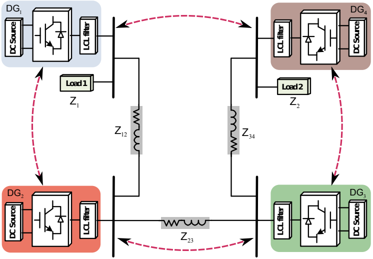

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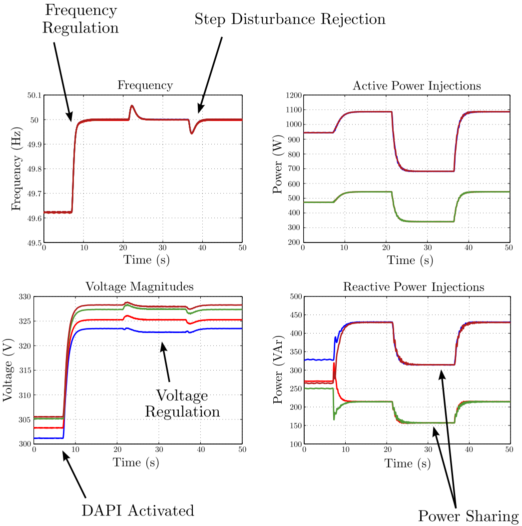
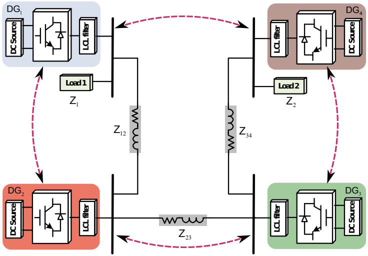


Summary

Distributed Inverter Control

- Primary control stability
- Distributed PI controllers
- Primary/tertiary connections
- Extensive validation



Future Work

- More detailed models
- More systematic designs
- $\mathcal{H}_2$ performance
- Monitoring $\Longleftrightarrow$ Feedback

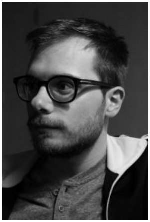
Acknowledgements

Florian Dörfler

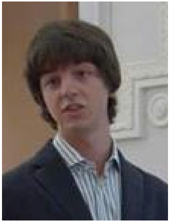
Francesco Bullo

Qobad Shafiee

Josep Guerrero



Marco Todescato

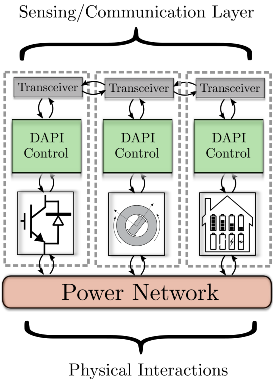


Basilio Gentile

Ruggero Carli

Sandro Zampieri

Question Time



http://engr.ucsb.edu/~johnwsimpsonporco/

jwsimpson@uwaterloo.ca

supplementary slides

An incomplete literature review of a busy field

ntwk with unknown disturbances $\cup$ integral control $\cup$ distributed averaging

- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcıa-Domınguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

DAPI Voltage Control – Performance [TIE '15]

$$\tau_i \dot{E}_i = -(E_i - E^*) - n_i Q_i(E) - e_i$$

$$\kappa_i \dot{e}_i = \beta_i(E_i - E_i^*) - {}_{j \subseteq \text{inverters}} \; b_{ij} \cdot \; \frac{Q_i}{Q_i^*} - \frac{Q_j}{Q_j^*}$$